DYNAMIC mini-CGETAX MODEL

14 July 2018: preliminary and not for quotation

The dynamic mini-CGETAX model is a new model designed to analyse the economic impacts of alternative tax policies. It can be characterised as an open economy Ramsey CGE model incorporating the Tobin-q theory of investment and heterogeneous consumers and businesses.

The new model is designed to complement the existing CGETAX model. CGETAX represents around 20 different significant taxes applied by federal, state and local governments, and the main behavioural responses to those taxes. This involves a fine level of detail, which includes 278 industries, nine types of capital, eight types of labour and three types of economic rents.

This fine level of detail on industries and taxes means CGETAX is useful in providing relatively refined estimates of the inefficiency costs of many taxes in the long run. However, tax policy is also influenced by other considerations including economic dynamics, equity and business size. The new model provides detail in these other areas. The idea is that, by using both CGETAX and dynamic mini-CGETAX to analyse tax policies, a wide range of policy considerations can be addressed.

The first area of detail in the dynamic mini-CGETAX model is dynamics. It takes into account capital stock adjustment costs in modelling the slow year-by-year response of capital to changes in tax policy. The resulting information on the time path the economy follows to a new equilibrium in responding to tax policy changes is of interest for three reasons.

First, there is a demand for information on the time paths. Providing such estimates can help widen understanding and acceptance of the long run results.

Second, dynamic mini-CGETAX can address the issue of whether using a dynamic model may lead to different policy recommendations for the long run than from using a long run equilibrium model such as CGETAX. The tax policy literature using Ramsey CGE models suggests that the recommendations for the long run are likely to be the same, but this is not always well understood.

Third, dynamics are fundamental in deciding how to transition changes to the taxation of capital income. Transition issues include whether new policies should be immediate or phased in and to what extent old policies should be grandfathered.

The second area of detail in the dynamic mini-CGETAX is equity. CGETAX distinguishes between low, mid and high income consumers, and takes into account the progressive nature of personal income tax. This allows the model to provide information at a broad level of detail on the equity impacts of tax policies.

The third area of detail in the dynamic mini-CGETAX model is business size. It distinguishes between larger companies, smaller companies and unincorporated businesses. These three types of businesses are each treated differently under the Australian corporate tax system. Larger companies are taxed at a higher rate (30 per cent) than smaller companies (27.5 per cent). Unincorporated businesses, which include sole traders, partnerships, family discretionary trusts and unit trusts such as real investment trusts, do not pay any company tax. Following the Differentiated Product Model of Gravelle and Kotlikoff (1993), in the model the three different types of businesses use identical production technology but produce differentiated products.

In the new model, expectations of companies and consumers are model-consistent, leading to forward-looking behaviour. For example, equity prices for each type of business will respond to the credible announcement of future changes in company tax policies. Similarly, consumer saving behaviour is influenced by expectations of future changes in consumption tax policies.

While CGETAX and the new model provide detail in different areas, they have similar core structures. They are both open economy Ramsey models. They recognise imperfect competition. They model substitution by businesses between labour and capital and by consumers between leisure and consumption. Key parameter values are aligned as closely as practical. The idea of this alignment is that results from both models can be combined to provide a broadly consistent picture for a wide range of economic impacts of tax policies.

Because the existing CGETAX model is relied upon for industry and tax detail, the new model treats these areas far more broadly. In place of 278 industries is a single non-housing industry, with the housing sector excluded from the model.

In place of around 20 detailed taxes in CGETAX, there are four broad taxes in the new model. These are taxes on labour income, consumption, company profits and a residual tax that is treated as lump sum in nature. The tax on labour income refers to personal income tax as it applies to labour incomes. The tax on consumption covers GST, excises, motor vehicle taxes and gambling taxes. The tax on corporate profits refers to company income tax. The model can also be used to simulate economic rent taxes based on cash flow that are often advocated. Following the literature, the lump sum tax is used as a benchmark in estimating the economic inefficiencies of the other taxes. This broad approach is useful in considering broad issues, particularly designing an optimal tax system.

The new model may be developed further in the future. One option is to distinguish taxes on asset incomes. In aggregate, asset incomes are taxed lightly due to the special tax treatments of owner-occupied housing, rental housing, superannuation and the system of franking credits. Another option is to introduce a housing sector. This would involve modelling the production of housing services and domestic portfolio choice between business and housing assets.

The modelling of businesses, consumers, government and markets are now explained in turn.

Businesses

In modelling business behaviour, there are three representative businesses. They are a larger company (superscript l), a smaller company (superscript m) and an unincorporated business (superscript l). While the general approach to modelling the three types of businesses is the same, their attributes are distinguished in setting the values for certain parameters. In particular, the three types of businesses may differ in the corporate tax rate they face, their eligibility to immediately expense investment, their ability to earn oligopoly rents and the likelihood that they engage in profit shifting. The general modelling approach is now presented, using the larger company for illustrative purposes.

Business behaviour

A representative company is an oligopolist who obtains oligopoly rents by engaging in mark-up pricing. Perfect competition is the polar case that can be modelled by setting the mark-up factor (*m*) to unity, but by default the mark-up factor is greater than unity.

The company chooses a plan for employment (*N*) and investment (*I*) to maximise the present value of its cash flow (*CF*), calculated as its cash flow before tax (*CFBT*) less company tax-related costs (*TKC*). In calculating this present value, the expected real annual discount rate (*r*) is able to vary from year-to-year. In the production function, output (*Y*) it produced with inputs of capital (*K*) and labour (*N*).

It is costly for the capital stock to grow at other than its equilibrium rate (*gr*), giving rise to sluggish adjustment of the capital stock based on the Tobin-q theory of investment. These adjustments costs are usually a little vaguely described as planning and installation costs, leaving it open to interpretation as to whether they would be allowable as a corporate tax deduction. This model allows for both possibilities (ω=0 or 1 for non-deductible or deductible).

This optimisation problem for the representative larger company is captured in the Lagrangian below. The standard capital accumulation identity, in which next year's capital stock is equal to this year's capital stock plus new investment less depreciation, is incorporated as a constraint. A second version of this constraint tracks the accumulation of the capital stock for tax purposes (*D*); it differs from the standard version by allowing for immediate expensing for tax purposes of a proportion of new investment[1]. This taxation version of the capital stock is referred to as the fiscal capital stock, to distinguish it from the physical capital stock.

All prices are measured relative to the price of aggregate output, which is therefore the numeraire. This aggregate output is a composite of the outputs of the three types of businesses, who sell their outputs at prices (*P*) that may differ.

$$\mathcal{L} = \sum_{u=t}^{\infty} \frac{1}{\prod_{i=0}^{u-t}\left(1 + r_{t+i}^{l}\right)} . \{CFBT_u^l - TKC_u^l + \mu_{u+1}^l . [(1-\delta). K_u^l + I_u^l - K_{u+1}^l] + \lambda_{u+1}^l . [(1-\delta). D_u^l + (1-\phi_u^l). I_u^l - D_{u+1}^l]\}$$

$$CFBT_u^l = P_u^l . f(K_u^l, N_u^l) - W_u . N_u^l - I_u^l - \frac{\psi}{2} . \frac{[I_u^l - (\delta + gr). K_u^l]^2}{K_u^l}$$

$$TKC_u^l = tkc_u^l . \left[P_u^l . f(K_u^l, N_u^l) - W_u . N_u^l - \phi_u^l . I_u^l - \delta . D_u^l - \omega . \frac{\psi}{2} . \frac{[I_u^l - (\delta + gr). K_u^l]^2}{K_u^l}\right]$$

[1] For simplicity, depreciation for tax purposes is assumed to be at replacement cost rather than historic cost.

Maximising this Lagrangian yields four first order conditions for employment, investment, the fiscal capital stock and the physical capital stock, as well as the constraints. After simplifying, these four first order conditions are as follows.

$$P_t^l.\frac{\partial f}{\partial N_t^l} = m_t^l.W_t$$

$$\frac{I_t^l}{K_t^l} = \delta + gr + \frac{1}{(1-\omega.tkc_t^l).\psi}.\left[\left(\mu_{t+1}^l + \left(1-\phi_t^l\right).\lambda_{t+1}^l\right) - \left(1-\phi_t^l.tkc_t^l\right)\right]$$

$$\lambda_t^l = \{(1-\delta).\lambda_{t+1}^l + \delta.tkc_t^l\}/\left(1+r_t^l\right)$$

$$\mu_t^l = \left\{\begin{matrix} (1-\delta).\mu_{t+1}^l + \left(1-tkc_t^l\right).\frac{P_t^l}{m_t^l}.\frac{\partial f}{\partial K_t^l} \\ + \left(1-\omega.tkc_t^l\right).\psi.(\delta+gr).\left[\frac{I_t^l}{K_t^l} - (\delta+gr)\right] \\ +\left(1-\omega.tkc_t^l\right).\frac{\psi}{2}.\left[\frac{I_t^l}{K_t^l} - (\delta+gr)\right]^2 \end{matrix}\right\} \Big/ \left(1+r_t^l\right)$$

Two of these conditions involve the marginal product of labour and capital. These are evaluated assuming that the production function follows the CES functional form. This leads to the seven equations describing the behaviour of the larger company, consisting of the production function, the four first order conditions from above, and the equations for the accumulation of physical and fiscal capital.

Production function for larger companies:

$$Y_t^l = \left[\left(ak.K_t^l\right)^{(\sigma-1)/\sigma} + \left(an_t.N_t^l\right)^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)} \qquad [1l]$$

Labour demand for larger companies (from marginal product of labour condition):

$$ln\left(N_t^l\right) = ln\left(Y_t^l/an_t\right) - \sigma.ln\left[\left(m_t^l.W_t\right)/\left(an_t.P_t^l\right)\right] \qquad [2l]$$

Investment demand for larger companies (Tobin-q):

$$\frac{I_t^l}{K_t^l} = \delta + gr + \frac{1}{\left(1-\omega.tkc_t^l\right).\psi}.\left[\mu_{t+1}^l + \left(1-\phi_t^l\right).\lambda_{t+1}^l + \phi_t^l.tkc_t^l - 1\right] \qquad [3l]$$

Shadow price of fiscal capital for larger companies (forward-looking):

$$\lambda_t^l = \{(1-\delta).\lambda_{t+1}^l + \delta.tkc_t^l\}/\left(1+r_t^l\right) \qquad [4l]$$

Shadow price of physical capital for larger companies (forward-looking):

$$\mu_t^l = \left\{ \begin{array}{c} (1-\delta).\mu_{t+1}^l + \left(1 - tkc_t^l\right).\frac{P_t^l}{m_t^l}.ak.\left[Y_t^l/\left(ak.K_t^l\right)\right]^{1/\sigma} \\ + \left(1 - \omega.tkc_t^l\right).\psi.(\delta + gr).\left[\frac{I_t^l}{K_t^l} - (\delta + gr)\right] \\ + \left(1 - \omega.tkc_t^l\right).\frac{\psi}{2}.\left[\frac{I_t^l}{K_t^l} - (\delta + gr)\right]^2 \end{array} \right\} \Big/ \left(1 + r_t^l\right) \qquad [5l]$$

Accumulation of physical capital for larger companies:

$$K_{t+1}^l = (1-\delta).K_t^l + I_t^l \qquad [6l]$$

Accumulation of fiscal capital for larger companies:

$$D_{t+1}^l = (1-\delta).D_t^l + \left(1 - \phi_t^l\right).I_t^l \qquad [7l]$$

Finally, when investment is such that capital stocks are not growing at their equilibrium rate of *gr*, adjustment costs are incurred as specified in the Lagrangian given earlier.

Capital stock adjustment costs for larger companies:

$$IA_t^l = \frac{\psi}{2}.\frac{\left[I_t^l - (\delta+gr).K_t^l\right]^2}{K_t^l} \qquad [8l]$$

Total supply

Following the Differentiated Product Model of Gravelle and Kotlikoff (1993), the total supply of output is a composite of the outputs of the three types of businesses. These outputs are imperfectly substitutable in a CES function.

Total output (supply):

$$Y_t = \left[\left(al.Y_t^l\right)^{(\sigma y-1)/\sigma y} + (am.Y_t^m)^{(\sigma y-1)/\sigma y} + (as.Y_t^s)^{(\sigma y-1)/\sigma y}\right]^{\sigma y/(\sigma y-1)} \qquad [9]$$

Minimising the costs of producing this total output gives demand equations for the outputs of both types of companies. These are inverted to determine the prices of these outputs. The price of total output does not appear in these equations because it is the numeraire.

Output demand for larger companies (solved for price):

$$ln\left(\frac{al.Y_t^l}{Y_t}\right) = -\sigma y.\,ln\left(\frac{P_t^l}{al}\right) \qquad [10l]$$

There are analogous output demand equations for smaller companies and for unincorporated businesses.

Profit shifting

Companies engage in profit shifting to reduce their costs inclusive of tax. If a proportion (*θ*) of profits is shifted to a tax haven, there is a tax saving equal to the amount of the tax base that is shifted (*θ.tkcov.BASE*), times the difference between the local tax rate and the tax haven tax rate (*tak-tkh*). Here, tkcov refers to the coverage of company profits by company income tax. This coverage factor is less than unity due, for example, to the deductibility of interest expenses.

Besides this saving tax cost, profit shifting also involves a tax avoidance cost. It is standard to assume that this rises with the product of the proportion that is shifted and the amount that is shifted. This captures the idea that profit shifting becomes more risky as the proportion of profits that is shifted rises.

Companies are assumed to maximise their net cost saving (*S*) from profit shifting, defined as the tax saving net of tax avoidance costs. The parameter *B* reflects the costliness of profit shifting, and is inversely proportional to it.

$$S = (tak - tkh).\theta.tkcov.BASE - \frac{1}{2.B}.\theta.\theta.tkcov.BASE$$

Choosing the proportion of profits that is shifted to maximise this saving gives the following simple solution. Profit shifting is proportional to the gap between the statutory tax rate and the tax rate in the tax haven.

$$\theta = B.(tak - tkh)$$

The value for θ affects both the effective company tax rate for local revenue collections (*tkr*) and the effective tax rate for investment decisions (*tkc*). The effective tax rate for revenue collections is simply the statutory tax rate (*tak*), adjusted down for the proportion of profits that is shifted and the profits coverage of company tax.

$$tkr = tkcov.(1 - \theta).tak$$

The effective tax rate for investment decisions is the statutory tax rate less the net cost saving from profit shifting (*S*) expressed as a proportion of the tax base (*BASE*), adjusted for the profits coverage of company tax. In deriving this result, the net cost saving expression is first simplified by using the solution for θ to eliminate *B*. The final formula for the effective tax rate for investment decisions is as follows.

$$tkc = tkcov.\left[tak - \frac{1}{2}\theta.(tak - tkh)\right]$$

The effective tax rate for investment decisions (*tkc*) is higher than the effective tax rate for local revenue collections ("tkr"). This difference reflects tax avoidance-related costs (*AVOID*) that add to the cost of investment.

$$AVOID = (tkc - tkr).BASE = tkh.\theta.BASE + \frac{1}{2}(tak - tkh).\theta.BASE$$

These tax avoidance-related costs are seen to have two components. The first component is the tax paid to the tax haven and the second component is cost of the avoidance activity. All of these tax avoidance-related costs are assumed to be incurred offshore, and thus represent a payment of income abroad.

While in practice some avoidance activity may occur locally, rather than offshore in the home country of the MNC and/or the tax haven, this does not change the outcome for consumer welfare. In the first case there is wastage of GDP on local tax avoidance activity while in the second case there is wastage of national income in paying for the same activity to be conducted offshore. Thus, it is harmless to simplify by assuming that all of the tax avoidance activity occurs offshore.

Profit shifting is modelled separately for the three types of businesses for two reasons. First, statutory company tax rates (*tak*) can differ according to business size, leading to different degrees of profit shifting. Second, profit shifting is likely to be more prevalent for larger companies than for smaller companies and unincorporated businesses, because profit shifting is associated with multinational corporations, who are often larger companies. This can be taken into account in setting different values for the profit shifting parameter (*B*).

The model equations for profit shifting by larger companies, which are based on the above workings, are set out below. There are analogous profit shifting equations for smaller companies and for unincorporated businesses.

Proportion of tax base shifted by larger companies:

$$\theta_t^l = B_t^l.\left(tak_t^l - tkh_t\right) \qquad [11l]$$

Effective tax rate for revenue-raising from larger companies:

$$tkr_t^l = tkcov.\left(1 - \theta_t^l\right).tak_t^l \qquad [12l]$$

Effective tax rate for investment decisions by larger companies:

$$tkc_t^l = tkcov.\left[tak_t^l - \tfrac{1}{2}\theta_t^l.\left(tak_t^l - tkh_t\right)\right] \qquad [13l]$$

Company tax base for larger companies:

$$BASE_t^l = P_t^l.Y_t^l - W_t.N_t^l - \phi_t^l.I_t^l - \delta.D_t^l - \omega.IA_t^l \qquad [14l]$$

Tax avoidance costs for larger companies:

$$AVOID_t^l = \left(tkc_t^l - tkr_t^l\right).BASE_t^l \qquad [15l]$$

Consumers

In modelling consumer behaviour, there are three representative consumers. They receive a low income (superscript *a*), a middle income (superscript *b*) and a high income (superscript *c*) respectively. Each type of consumer has the same utility function. They differ in the wage attracted by the type of labour they provide and their initial holdings of financial assets. They also differ in the average tax rate they face on their labour income because the model takes into account the progressive nature of the personal income tax system. The general modelling approach is now presented, using the low income consumer (superscript *a*) for illustrative purposes.

Consumer behaviour

The modelling of consumers is taken from the well-known Ramsey model of the economy. In that model, a consumer behaves as a dynasty, concerned about the utility levels of itself and its descendants to an infinite time horizon. The Ramsey model is the basis of some key propositions in the optimal tax literature.

This optimisation problem for the low income (superscript *a*) representative consumer is captured in the Lagrangian below. All amounts are expressed on a per consumer basis. The representative consumer maximises its discounted future utility. In any year, it gains utility *(u)* from both its consumption *(hc)* and its leisure time, which is equal to its available time (normalised to unity) less the time it spends employed (*n*).

In aggregating its utility over the years, the willingness of the representative consumer to substitute utility between years is limited, so utility in each year is raised to the power of a factor (β) that is less than unity. Further, because the representative consumer cares about its dynasty, its utility level grows with the rate of population growth (*pop*). At the same time, the representative consumer discounts the future using a rate of time preference (ρ).

The Lagrangian also includes the intertemporal budget constraint facing the representative consumer. In a given year, it can accumulate assets if it saves, that is, if its income exceeds its consumption, after tax payments are taken into consideration.

The consumer has two sources of income. The first source is ex ante asset income from the part of the capital stock that is owned domestically rather than by foreigners *(r.E.kd)*. The second source is labour income (*W.n)*, net of labour income tax that is applied progressively by exempting a threshold amount *(tmn.(W.n-th))*. For solving the consumer problem, it is convenient to re-write this as the difference between potential labour income if no leisure time were taken *(W-tmn.(W-th))* and the opportunity cost of leisure time *((1-tmn).W.(1-n))*.

Out of this income, the consumer funds consumption inclusive of a consumption tax *((1+tac).hc)* and payment of a lump sum tax *(tl.W)*. The residual amount is saving, which is used to increase the value of the consumer's stake in the capital stock $(Ed_{t+1}.kd_{t+1}–Ed_t.kd_t)$. Because this increase is expressed in per consumer terms, it is reduced by the population growth factor *(1+pop)*.

Consumer problem:

$$\mathcal{L} = \sum_{q=t}^{\infty} \frac{(1+pop)^{q-t}}{(1+\rho)^{q-t}} . \Big\{ u\big(hc_q^a, 1-n_q^a\big)^{\beta} / \beta$$
$$+ \lambda_q^a . \big[\big(1+r_q\big). Ed_q . kd_q^a + \big(1-tmn_q\big). W_q^a + tmn_q . th_q$$
$$- \big(1-tmn_q\big). W_q^a . \big(1-n_q^a\big) - tal_q . W_q^a - \big(1+tac_q\big). hc_q^a$$
$$- (1+pop). Ed_{q+1} . kd_{q+1}^a \big] \Big\}$$

Maximising this Lagrangian yields the four equations below, made up of the budget constraint and the first order conditions for asset holdings (the Euler equation), consumption and leisure.

$$\begin{aligned}(1+pop).Ed_{t+1}.kd_{t+1}^{a} \\ &= (1+r_t).Ed_t.kd_t^a + (1-tmn_t).W_t^a + tmn_t.th_t - tal_t.W_t^a \\ &- (1-tmn_t).W_t^a.(1-n_t^a) - (1+tac_t).hc_t^a\end{aligned}$$

$$\lambda_{t-1}^{a} = \frac{1+r_t}{1+\rho}.\lambda_t^a$$

$$u(hc_t^a, 1-n_t^a)^{\beta-1}.\frac{\partial u}{hc_t^a} = \lambda_t^a.(1+tac_t)$$

$$u(hc_t^a, 1-n_t^a)^{\beta-1}.\frac{\partial u}{\partial(1-n_t^a)} = \lambda_t^a.(1-tmn_t).W_t^a$$

For use in the model, these equations require further manipulation.

The budget constraint needs to be converted from that of the representative consumer to that of all consumers aggregated together. Amounts per consumer were represented in the Lagrangian in lower case. The corresponding aggregate amounts are represented in upper case and are calculated by simply multiplying the amount per consumer by the number of consumers (*H*). One effect of this conversion is to eliminate the population growth factor.

$$\begin{aligned}Ed_{t+1}.KD_{t+1}^a &= (1+r_t).Ed_t.KD_t^a + (1-tmn_t).W_t^a.H_t^a + tmn_t.th_t.H_t^a \\ &- tal_t.W_t^a.H_t^a - (1-tmn_t).W_t^a.(H_t^a - N_t^a) - (1+tac_t).HC_t^a\end{aligned}$$

The budget constraint in the Lagrangian refers to ex ante asset income calculated using the opportunity cost of funds *(r)*, as is appropriate in consumer planning. However, when shocks are applied to the model, asset income may be different ex poste than ex ante, which needs to be taken into account. This involves replacing the expected return with the actual return made up of dividend payments and capital gains.

$$r_t.Ed_t.KD_t^a = (KD_t^a/KD_t).DIVD_t + (Ed_{t+1} - Ed_t).KD_t^a$$

Making this substitution and simplifying gives the final consumer budget constraint appearing in the model. It shows how domestic equity in the capital stock increases with domestic saving.

Accumulation of domestic equity for low-income consumer:

$$\begin{aligned}&Ed_{t+1}.(KD_{t+1}^a - KD_t^a) = (KD_t^a/KD_t).DIVD_t + (1-tmn_t).W_t^a.N_t^a + \\ &tmn_t.th_t.H_t^a - tal_t.W_t^a.H_t^a - (1+tac_t).HC_t^a\end{aligned} \quad [16a]$$

In using the remaining three equations, a CES function is used in modelling the dependence of the contemporaneous utility of the representative consumer on consumption and leisure.

$$u_t^a = \left[(ac.hc_t^a)^{(\sigma u-1)/\sigma u} + \left(ale_t.(1-n_t^a)\right)^{(\sigma u-1)/\sigma u}\right]^{\sigma u/(\sigma u-1)}$$

The corresponding constant utility price index (PU) is as follows.

Contemporaneous constant utility price index for low-income consumer:

$$PU_t^a = \left[\left(\frac{1+tac_t}{ac}\right)^{1-\sigma u} + \left(\frac{(1-tmn_t).W_t^a}{ale_t}\right)^{1-\sigma u}\right]^{1/(1-\sigma u)} \quad [17a]$$

The first order conditions for consumption and leisure can be solved for the Lagrange multiplier.

$$\lambda_t^a = \frac{{u_t^a}^{\beta-1}}{PU_t^a}$$

Using this solution in the Euler equation and simplifying gives the expression for the growth rate in contemporaneous utility, which involves the elasticity of intertemporal substitution *(σt=1/(1-β))*.

Euler equation for contemporaneous utility (forward-looking) for low-income consumer:

$$ln\left[\frac{u_{t+1}^a}{u_t^a}\right] = \sigma t.\left\{ln\left[\frac{1+r_{t+1}}{1+\rho}\right] - ln\left[\frac{PU_{t+1}^a}{PU_t^a}\right]\right\} \quad [18a]$$

This can be compared with the standard Euler equation. The price of utility (relative to the price of output, our numeraire) is stable in the steady state under our default assumptions. Putting this term to one side, to a linear approximation we have the standard Euler equation. That is, growth in the contemporaneous utility of the representative consumer is equal to the elasticity of intertemporal substitution times the real rate of return net of the rate of time preference. In calibrating the model, the rate of time preference is set so that growth in the contemporaneous utility of the representative consumer matches growth in labour productivity, and so is sustainable. This is the knife-edge solution that is widely used when applying the Ramsey model in a small open economy.

Inserting the solution for the Lagrange multiplier into first order conditions for consumption and leisure and simplifying provides the associated demand equations. In writing the demand for leisure, the potential labour supply of the representative consumer was normalised to unity. Hence, the total potential labour supply of consumers is *H*.

Consumption demand:

$$ln[ac.HC_t^a] = -\sigma u.\,ln\left[\frac{(1+tac_t)/ac}{PU_t^a}\right] + ln[u_t^a.H_t^a] \quad [19a]$$

Leisure demand (labour supply):

$$ln[ale_t.(H_t^a - N_t^a)] = -\sigma u.\,ln\left[\frac{(1-tmn_t).W_t^a/ale_t}{PU_t^a}\right] + ln[u_t^a.H_t^a] \quad [20a]$$

From the expression for discounted future utility appearing in the Lagrangian, we can derive the following equation for how consumer welfare evolves over time.

Consumer welfare (forward-looking):

$$V_t^a = [\sigma t/(\sigma t - 1)]. H_t^a . u_t^{a(\sigma t-1)/\sigma t} + \frac{1}{1+\rho} . V_{t+1}^a \qquad [21a]$$

Equations [16] to [21] refer to low income consumers (superscript *a*). They provide the five equations needed to solve for the prices and volumes of contemporaneous utility *(P, u)*, consumer welfare *(V)*, consumption *(C)* and leisure *(H-N)*. Labour supply can be recovered as available time *(H)* less leisure *(H-N)*. There are analogous equations for middle income consumers (superscript *b*) and high income consumers (superscript *c*).

Government

The government funds its demand for output by raising four taxes. These taxes are company tax, consumption tax, labour income tax and a lump sum tax.

In modelling company tax, the model takes into account that company tax is applied at different rates to the three types of businesses. Equation [22l] is the equation for company tax collected from larger companies.

Company tax from larger companies:

$$TK_t^l = tkr_t^l . BASE_t^l \qquad [22l]$$

Total company tax consists of the sum of collections from the three types of businesses. Within this, collections from unincorporated businesses are zero.

Company tax:

$$TK_t = TK_t^l + TK_t^m + TK_t^s \qquad [23]$$

In modelling personal income tax collected from labour income, the model takes into account the progressive nature of the personal income tax scale. This is done using the common approximation that the personal income tax scale is linear. That is, personal income tax is applied at a single marginal rate (*tmn*) above a threshold level of income (*th*).

One option would be to specify the marginal rate and the threshold income as exogenous variables. Instead, these are endogenous variables derived from underlying exogenous policy for the average of tax (*tan*) of tax and a progressivity factor (*tprog*). This approach avoids the unending, automatic bracket creep that would occur if the threshold income were exogenous, and makes changing equity policy more intuitive.

Under an exogenous average rate of labour income tax, labour income tax collections (*TN*) are modelled simply as the average tax rate (*tan*) applied to total labour income (*YN*).

Labour income tax:

$$TN_t = tan_t . YN_t \qquad [24]$$

The progressivity factor (*tprog*) determines the ratio of the marginal rate of tax (*tmn*) to the average rate of tax (*tan*). To apply this approach to approximate the actual tax scale, the progressivity factor (*tprog*) is set equal to the elasticity of tax collections with respect to income per taxpayer.

Marginal rate of labour income tax:

$$tmn_t = tprog_t . tan_t \qquad [25]$$

Finally, the threshold income for personal income tax (*th*) is set relative to average labour income per consumer (*WH*) to achieve the exogenously determined progressivity factor (*tprog*).

Threshold for labour income tax:

$$th_t = [(tprog_t - 1)/tprog_t] . WH_t \qquad [26]$$

Consumption tax collections are equal to the average rate of consumption tax applied to total consumption, which is aggregated over the three types of consumers.

Consumption tax:

$$TC_t = tac_t . \left(HC_t^a + HC_t^b + HC_t^c\right) \qquad [27]$$

Revenue from lump sum tax is set to automatically balance the government budget.

Lump sum tax:

$$TL_t = G_t - (TK_t + TN_t + TC_t) \qquad [28]$$

This revenue target for lump sum tax is achieved by automatically adjusting its tax rate:

$$tal_t = TL_t / \left(W_t^a . H_t^a + W_t^b . H_t^b + W_t^c . H_t^c\right) \qquad [29]$$

This modelling of lump sum tax assumes that the three types of consumers are taxed independently of their employment decisions but proportionally to their wage rate. The tax liability must be independent of employment decisions for it to be lump sum in nature. It is set proportional to each type of consumer's wage rate, rather than as a simple head tax, to avoid artificial and implausible equity effects when an increase (reduction) in lump sum tax is used to fund a reduction (increase) in one of the other three taxes.

The model can also be used to model both source and destination-based cash flow taxes. The corporate tax can be converted to a source-based cash flow tax by specifying full, immediate expensing of investment *(Ø=1)*. By default, depreciation allowances would continue to be provided for old capital. The corporate tax can be converted to a destination-based cash flow tax by setting the corporate tax rate to zero *(tak=0)* and increasing the rate of tax on consumption *(tac)* and reducing the average and marginal rates of tax on labour income *(tan* and *tmn)* by the same amount.. By default, depreciation allowances would not continue to be provided for old capital.

Markets

Types of Labour

In the modelling of business behaviour, equations were derived for labour demand by each type of business. Total demand for labour is obtained by summing these labour demands.

Labour Market demand:

$$Nd_t = N_t^l + N_t^m + N_t^s \qquad [30]$$

To meet this labour demand, businesses can draw on the three types of labour supplied by households. These three types of labour are assumed to be imperfectly substitutable in production. The optimal mix of labour demand between the three types is obtained by minimising labour costs (*YN*) subject to the employment requirement (*N*).

Labour bill:

$$YN_t = W_t^a . Nd_t^a + W_t^b . Nd_t^b + W_t^c . Nd_t^c \qquad [31]$$

Employment requirement:

$$Nd_t = \left[(ba.Nd_t^a)^{(\sigma n-1)/\sigma n} + \left(bb.Nd_t^b\right)^{(\sigma n-1)/\sigma n} + (bc.Nd_t^c)^{(\sigma n-1)/\sigma n}\right]^{\sigma n/(\sigma n-1)}$$

This cost minimisation exercise provides a set of equations for the demand for labour of each type. To illustrative this, the demand equation for low-skilled labour is presented below.

Demand for labour of type a:

$$ln(Nd_t^a) = ln(Nd_t/ba) - \sigma n. ln((W_t^a/ba)/W_t) \qquad [32a]$$

The cost minimisation exercise also provides an overall wage cost index. This wage index depends on the wage for each type of labour and the parameters of the employment requirement relationship.

Wage index:

$$W_t = \left[(W_t^a/ba)^{1-\sigma n} + \left(W_t^b/bb\right)^{1-\sigma n} + (W_t^c/bc)^{1-\sigma n}\right]^{1/(1-\sigma n)} \qquad [33]$$

This wage index was used in modelling business behaviour earlier.

The model also keeps track of total employment by simple addition of the demand for the three labour types. This is for reporting purposes only.

Employment:

$$N_t = Nd_t^a + Nd_t^b + Nd_t^c \qquad [34]$$

For the earlier modelling of the personal income tax threshold (*th*), average labour income per consumer is needed.

Average labour income per consumer.

$$WH_t = YH_t/H_t \qquad [35]$$

So far we have independently modelled demand and supply for each of the three types of labour. However, wages of each type adjust to satisfy the labour market clearing condition of demand equals supply. This clearing condition is illustrated using low-skilled labour.

Labour market equilibrium for low skilled labour (achieved through adjustment of the wage):

$$Nd_t^a = Ns_t^a \qquad [36a]$$

Other Markets

For the goods market to be in equilibrium, the composite supply of total output from the three types of businesses must equal the total demand. Demand includes consumption by the three types of consumers, investment demand by the three types of businesses and the associated capital stock adjustment costs, government demand (*G*) and net exports (*NX*).

Goods Market equilibrium (solved for net exports):

$$Y_t = HC_t^a + HC_t^b + HC_t^c + I_t^l + I_t^m + I_t^s + IA_t^l + IA_t^m + IA_t^s + G_t + NX_t \qquad [37]$$

It is assumed that Australia is a small open economy in the world capital market. The marginal foreign investor in Australian companies of a given size requires the same post-company tax rate of return that is available elsewhere in the world. This is illustrated below for larger companies.

Real annual discount rate for investment in larger companies:

$$r_t^l = r_t \qquad [38a]$$

The same assumption is made for smaller companies and unincorporated businesses. While it can be objected that foreign investment in these smaller entities is rare, foreign investors indirectly influence the cost of capital for these smaller entities through two channels. First, domestic investors undertake portfolio substitution between investing in larger companies and the smaller entities, introducing a link between their costs of capital. Second, smaller entities access the banking system, which may be financed at the margin by foreign investors.

Businesses generate cash flow through their cash profits and equity raisings. In the model cash flow is paid out in full as dividends because retained earnings and debt are not taken into account. The dividend equation is illustrated using larger companies.

$$DIV_t^l = P_t^l.Y_t^l - W_t.N_t^l - I_t^l - \frac{\psi}{2}.\frac{\left[I_t^l - (\delta + gr).K_t^l\right]^2}{K_t^l} - tkc_t^l.BASE_t^l + E_{t+1}^l.\left(K_{t+1}^l - K_t^l\right) \qquad [39l]$$

The required rate of return on domestic equity is determined abroad under the open economy assumption of equation [38]. Hence, the following condition sets the expected return on domestic equity this required return. The expected return on domestic equity consists of an expected capital gain and the dividend yield.

$$r_t^l = \left(E_{t+1}^l - E_t^l\right)/E_t^l + \frac{DIV_t^l}{E_t^l.K_t^l}$$

In the short-term, in flexible, forward-looking asset markets, the price of equity will adjust to ensure this condition holds. Hence, in the model, the condition is inverted to determine the price of equity.

Equity market for larger companies (forward-looking):

$$E_t^l = \left(E_{t+1}^l + \frac{DIV_t^l}{K_t^l}\right)/\left(1 + r_t^l\right) \quad [40l]$$

Various aggregates can now be calculated by aggregating over the three types of businesses. These aggregates are the total value of dividend payments to domestic investors, the total capital stock, the domestically-owned capital stock and the average price of equity for that stock.

Dividends for domestic investors:

$$DIVd_t = \left(1 - KF_t/K_t^l\right).DIV_t^l + DIV_t^m + DIV_t^s \quad [41]$$

Aggregate capital:

$$K_t = K_t^l + K_t^m + K_t^s \quad [42]$$

Domestically-owned capital:

$$KD_t = KD_t^a + KD_t^b + KD_t^c \quad [43]$$

Average Price of Domestically-owned equity:

$$Ed_t = \left(E_t^l.\left(K_t^l - KF_t\right) + E_t^m.K_t^m + E_t^s.K_t^s\right)/KD_t \quad [44]$$

Having determined the total capital stock (K) and domestic ownership of it (KD), foreign ownership can be determined residually. However, we prefer to model foreign ownership independently. Specifically, foreign equity accumulates to fund the current account deficit.

Accumulation of foreign equity:

$$E_{t+1}^l.(KF_{t+1} - KF_t) = \left(KF_t/K_t^l\right).DIV_t^l - NX_t + AVOID_t^l + AVOID_t^m + AVOID_t^s \quad [45]$$

We then check whether the capital stock is equal to the sum of the domestic and foreign-owned portions. This is a useful cross-check on the coding of the model.

Capital cross-check equation:

$$KCHECK_t = K_t^l + K_t^m + K_t^s - \left(KD_t + KF_t\right) \quad [46]$$

Long-run growth identities

The long run rate of economic growth is exogenous to the model. It is driven by population growth at the rate *pop* and labour efficiency growth at the rate *prod*.

Population aged 15-64 increase at rate *pop* for consumers of type a:

$$H_t^a = (1 + pop).H_{t-1}^a \qquad [47a]$$

$$H_t = H_t^a + H_t^b + H_t^c \qquad [48]$$

Labour efficiency growth in work and leisure at rate *prod*:

$$an_t = (1 + prod).an_{t-1} \qquad [49]$$

$$ale_t = (1 + prod).ale_{t-1} \qquad [50]$$

Sustainable economic growth rate (used in modelling investment):

$$gr = (1 + pop).(1 + prod) - 1 \qquad [51]$$

Government spending growth:

$$g_t = (1 + gr).g_{t-1} \qquad [52]$$

Key Parameters

Code	Value	Parameter Description
σ	0.8	Elasticity of substitution between labour and capital
σy	3	Elasticity of substitution between outputs of the three types of businesses
σt	1/3	Elasticity of intertemporal substitution
σn	2	Elasticity of substitution between the three labour types
σu	1.5	Elasticity of substitution for consumption and leisure
r	0.073	Annual real world rate of return on capital
ρ	0.028	Annual rate of time discount
δ	0.067	Rate of annual depreciation of business capital
ω	1	Adjustment costs (0=non-deductible,1=deductible)
pop	0.014	Rate of population growth
prod	0.014	Rate of labour efficiency growth

Exogenous variables

Variable Code	Variable Description
tak_t^l	Statutory rate of company tax for larger companies
tak_t^m	Statutory rate of company tax for smaller companies
tak_t^s	Statutory rate of company tax for unincorporated businesses
ϕ_t^l	Proportion of investment immediately tax expensed for larger companies
ϕ_t^m	Proportion of investment immediately tax expensed for smaller companies
ϕ_t^s	Proportion of investment immediately tax expensed for uninc. businesses
B_t^l	Profit shifting parameter for larger companies
B_t^m	Profit shifting parameter for smaller companies
B_t^s	Profit shifting parameter for unincorporated businesses
m_t^l	Price mark-up factor for larger companies
m_t^m	Price mark-up factor for smaller companies
m_t^s	Price mark-up factor for unincorporated businesses
th_t	Tax rate in tax haven
$tkcov_t$	Rate of tax on labour income
tan_t	Average rate of tax on labour income
$tprog_t$	Progressivity factor for tax on labour income
tac_t	Rate of tax on consumption
r_t	Required rate of return to capital
$time_t$	Time trend